Aircraft Performance Modeling with Polynomial Function using Small Variable Units Technique

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Contribution to the State of the Art

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Abstract: Mathematical methods of Regression analysis, with focus on polynomial regression, are useful analytical methods for trend-line definitions of an aircraft aerodynamics. Nomogram is graphical interpretation of polynomial regression analysis results and aero-dynamical performance according to different environmental parameters and requirements. If diagram defines relations of two variables, where the one is dependable of another one (*y=f (x)*), the nomogram defines relations among three variables, where the one is resulting and dependable of another two undependable. The Small Variable Units Technique is efficient method to transfer nomograms' data in polynomial equitation which can give us different mathematical models of an aircraft performance. In combination with time based navigation, digitalization of aerodynamical characteristic will be a step forward to Continuous Climb and Descent Trajectories, as the most optimal one.

Keywords: performance optimization, digitalization, composite, regression, nomogram.

Introduction

Fast technical development of new age require more efficient and intelligent way of transportation what put air-transport, air industry and aeronautical science in focus, as solution for modern business models.

Introduction of ultra-light composite materials, efficient and lighter electric propulsions and digitalization of onboard electronics, have brought various alternative aircraft constructions like quadricopters and ultra-light airplanes.

Modern standards of environmental protection, energy efficiency and tendencies for developments of air-traffic networks over large urban area for commuter and personal transportation, put in focus two main aircraft performance requests which could be defined as "On-spot landing" and energy efficient aero-dynamical performance [1].

Even from the beginning of aeronautical science an aircraft aero-dynamical performance was the main focusing issue. Earth gravity and air as medium which provides lift, drag and propulsion force of an aircraft make mathematical modeling a complex task with many dependable and undependable elements and functions.

Mathematical methods of Regression analysis [2], with focus on polynomial regression, are useful analytical methods of experimental testing of an aircraft and definition of trend-line for physical characteristics of aerodynamics which are important for further aircraft exploitation.

Nomogram is graphical interpretation of polynomial regression analysis results and aero-dynamical performance according to different environmental parameters and requirements for aircraft exploitation [3]. Comparing to the diagram, nomogram is mathematical model which presents dependencies of more than two variables, with capability to connect it selves with other nomograms in complex system what gives us mathematical model of aerodynamical characteristic of an aircraft [4].

The Small Variable Units Technique is efficient method by which nomograms' data can be transferred in polynomial equitation and give us mathematical models of an aircraft performance efficient for real time exploitation with various possibilities

for aircraft performance software design according to actual needs and requirements.

The Small Variable Units Technique

If diagram defines relations of two variables, where the one is dependable of another one ($y=f(x)$), the nomogram defines relations among three variables, where the one is resulting and dependable of another two undependable. Mathematical expression of this definition could be presented as following:

$$y=f(x) \wedge f(z)$$

Essentially, the nomogram has three-dimensional nature what is also the biggest problem when it has to be presented in two-dimensional space, like sheet of paper. The solution was found in the way that the first undependable variable can be presented at x-axis, while the cardinal (specific) values of the second one are selected and drown as a series of diagrams or lines with, in most of cases, variable displacement (Figure 1).

The values of the nomogram lines, usually, are forming the polynomial lines (parabolas) which are changing its shape according to the physical lows. Determinations of the polynomial equitation which are capable to describe those laws are the primary task of the data digitalization process. For the purpose of this research MS Excel functions will be used as operational tool.

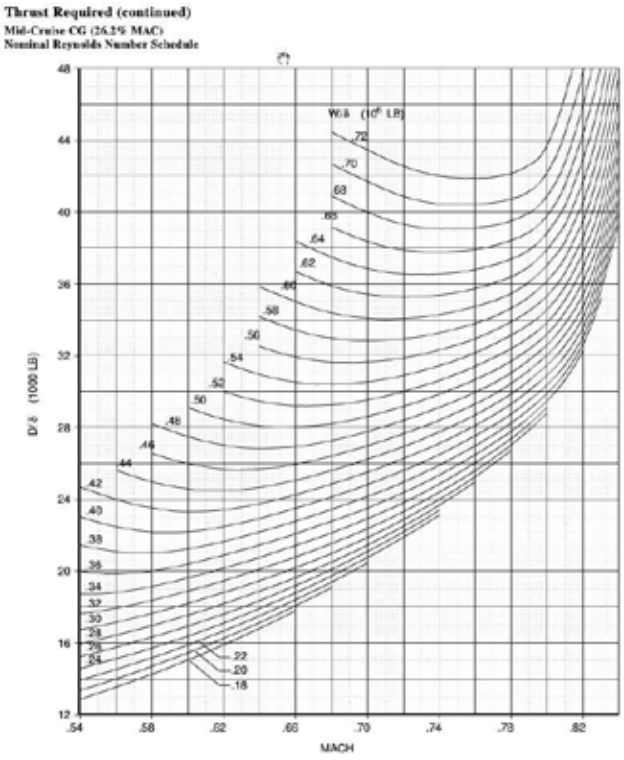

Figure 1

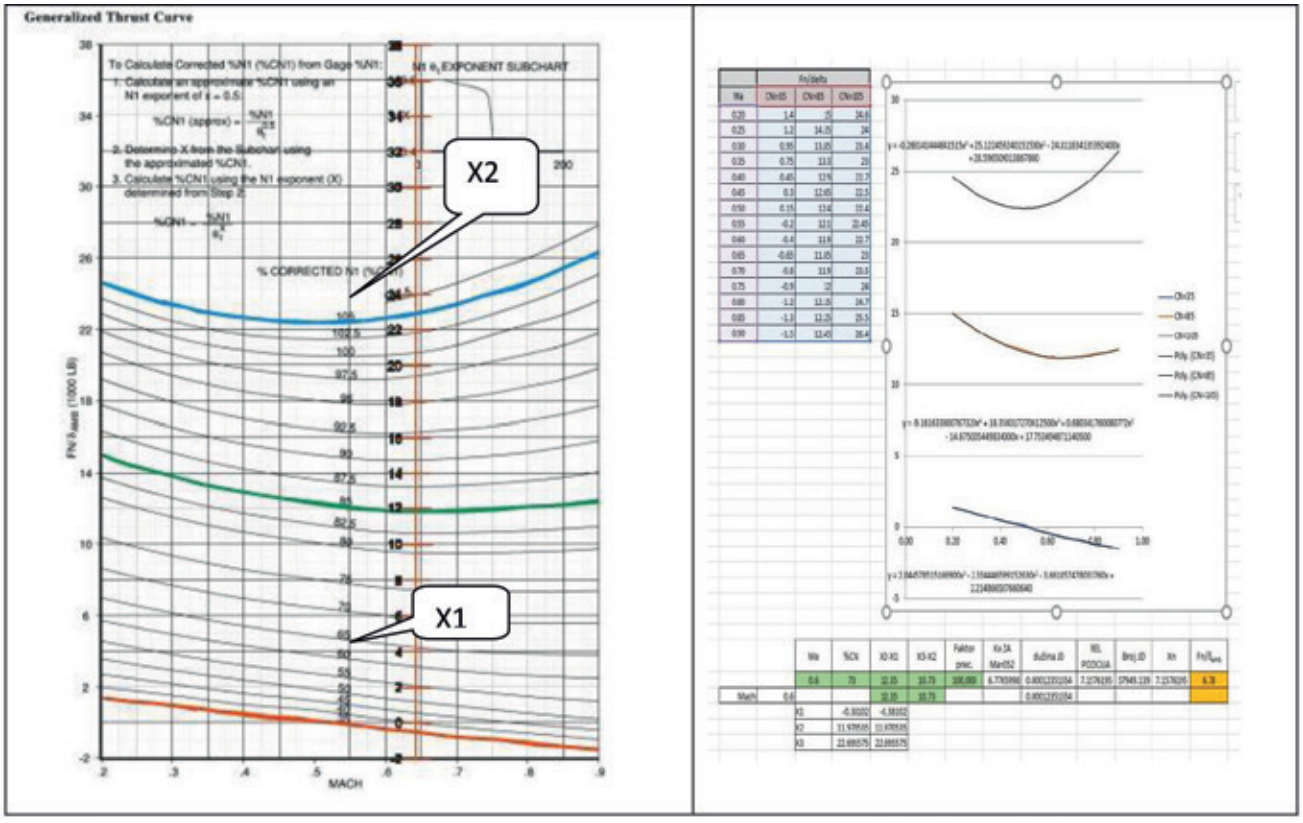

Figure 2. *Nomogram importation in Excel*

Digitalization of a standalone nomogram (three-dimensional) case

The small units with the variable length which are reflecting the variations of the bordering polynomial lines are the core of this technique.

For the successful digitalization process geometrical characteristics of the nomogram polynomial lines have to be carefully evaluated and concrete action plan must be defined, before the digitalization technique should be applied. This can be accomplished if we make the table with data from the nomogram polynomial bordering lines and form the nomogram lines area (Figure 2).

For the nomogram bordering lines the polynomial expression can be defined by the Microsoft Excel "Add Trendline" function, and the polynomial formula looks like this:

$y = 2{,}044578515166900x^3 - 2{,}334446599152630x^2 - 3{,}661857478033760x + 2{,}214866587660640$

Next step implies the relations definition between the resulting values of the function (dependable variable) and the second undependable variable. This can be established if separate table is made with the values of the cardinal lines for the specific value from the x-axis.

When the "y-z" diagram is formed, the law of the polynomial lines displacement variable can also be defined by the "Excel Add Trendline" function (Figure 3).

The third step includes polynomial lines amount calculation that the digitalized nomogram will include. If the characteristic value from x-axis enters two polynomial bordering lines formulas the difference between resulting values will be the length of the straight line between the X2 and X1 points. If this straight line is divided with infinity number the absolute precision of the techniques will be achieved and possibility to calculate the values of nomogram with infinite decimal places will be achieved.

However, with the more pragmatic approach, due to practical use, resulting straight line between bordering points could be divided with determined large number (Precision factor like 100.000) which will define the final value of the small unit that could be used in following calculations with required level of approximation. As we "walk" through the domain on the x-axis the length of the small unit will be

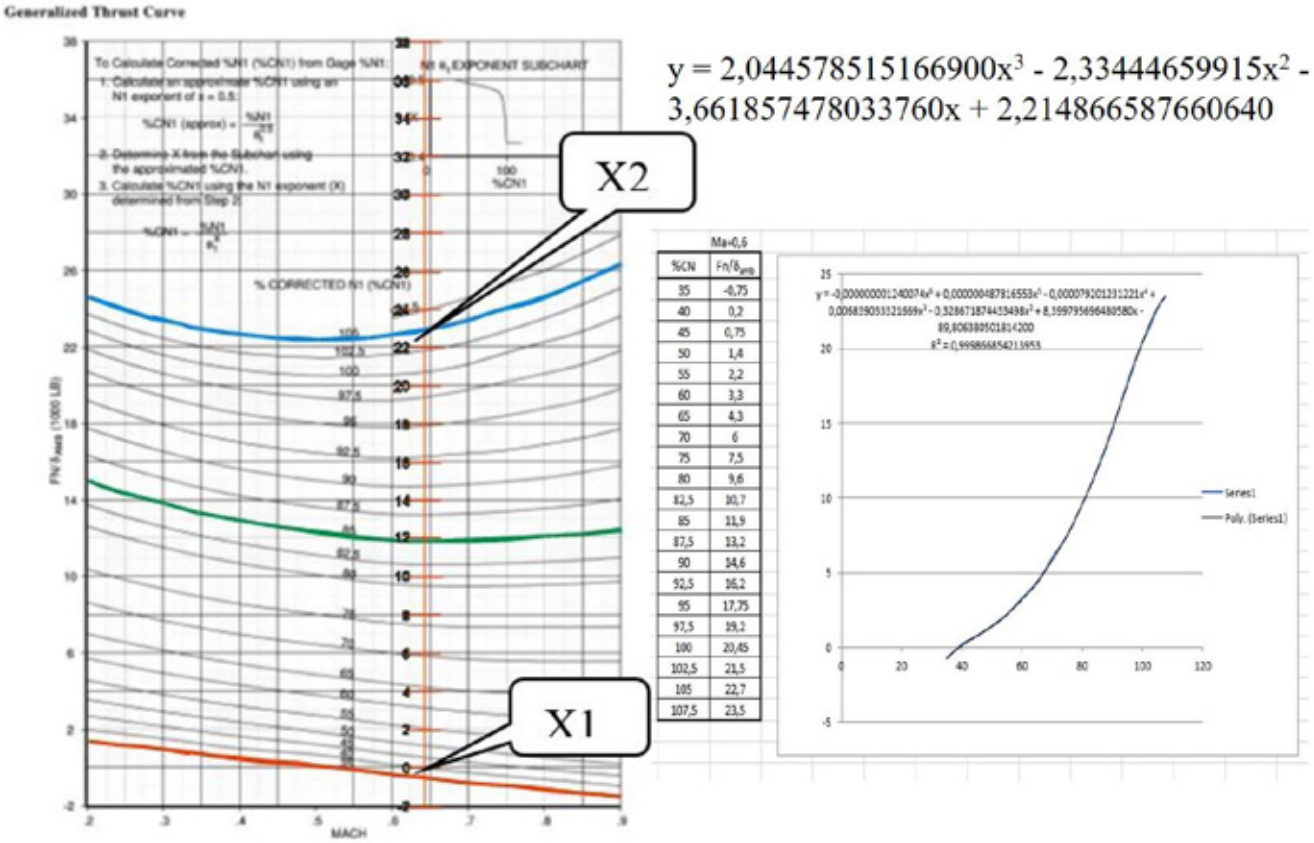

Figure 3. Polynomial lines displacement calculations

		Ma	%CN	X2-X1	X3-X2	Faktor prec.	Kx ZA Ma=052	dužina JD	REL POZICIJA	Broj JD	Xn	Fn/δ_{amb}
		0.6	73	12.35	10.73	100,000	6.7765998	0.00012351554	7.1576195	57949.139	7.1576195	6.78
Mach	0.6			12.35	10.73			0.00012351554				
		X1	-0.38102	-0.38102								
		X2	11.970535	11.970535								
		X3	22.695575	22.695575								

Figure 4. *Nomogram data digitalization calculator*

changed and ruled by the parallel point of the nomogram bordering lines, what gives us "The Small Variable Unit".

The fourth step implies the straight-line length calculation for the characteristic second independent variable values. The polynomial formula, defined in step two, can determine the position of the point, for the characteristic previously defined x-axis value that exactly one and required parabola is passing through. If the length of this straight line is divided by the length of the Small Variable Unit, the result will be the number of the small unit for the required value of the variable two. Multiplication of the Small Variable Unit value or number with its length for the required value from the x-axis domain will produce the set of points and exact shape of the required polynomial line.

System of nomograms digitalization (multi-dimensional case)

As a difference, in this case there are three undependable entering variables which are related to the fourth resulting dependable one, what mathematically can be presented as:

$$y=f(x) \wedge f(z) \wedge f(k)$$

In this case, the values of the first variable are shown at x-axis, while another two are defined as the specific cardinal values. Practically in this situation result is expressed as the series of connected nomograms according to the list of specific values of

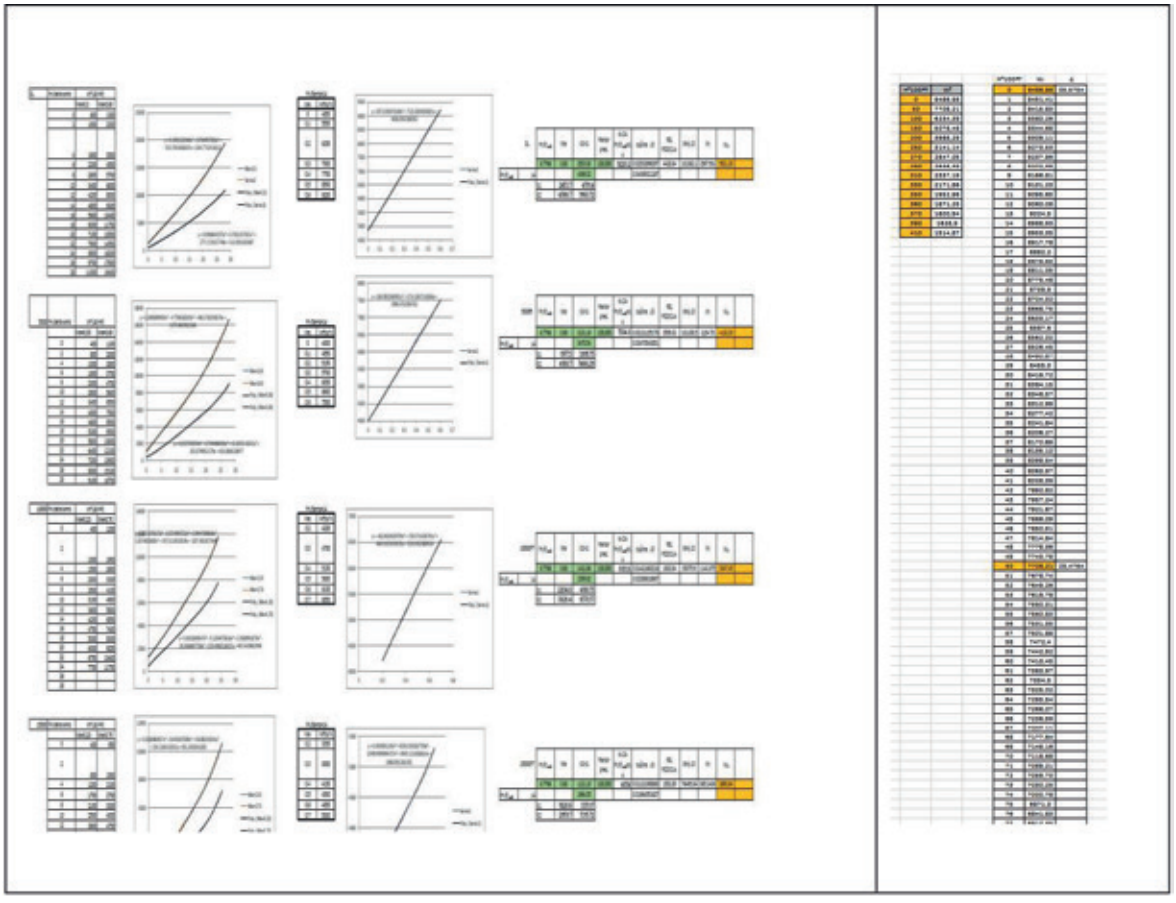

Figure 5. *System of nomograms and the dynamic database*

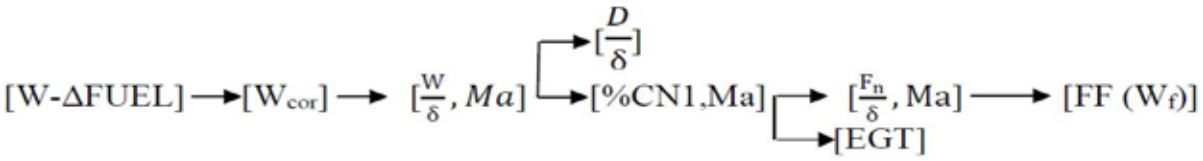

Figure 6. *Performance data calculations algorithm.*

the third variable.

In digitalization process every single nomogram will be separately digitalized in Excel in the way that for the same entering values of the first and second entering variable the result will be the set of data for all specific values of the third variable.

The characteristic Data Base table will be formed from the resulting set of data, which will be stretched out in a spectrum of resulting values for the appropriate level of decimal places (or decade units) of the third entering variable. The values between the resulting data will be proportionally filled and the specific resulting dynamic data base will be formed. The Excel "VLOOKUP" or "HLOOKUP" functions will be used to search dynamic database according to the requested criteria of the third entering variable.

Small Variable Units Technique approximations control

Aside of the fact that SVUT digitalization technique is very precise it also allows the control of approximations and errors which can be made during the process, due to various reasons. Using the "IF/AND" Excel functions every domain of the entering variables can be divided at maximum six separate co-domains. This implies that every single nomogram's three-dimensional space could be divided to the maximum of the separately digitalized 36 parts, in case that «MS Excel» is used as tool for calculations.

The second way of approximations control technique implies that set of testing data will be formed by which the level of resulting data deviations will be determined. According to the deviations data the correction function will be defined and the corrections will be in-calculated in the final results.

Mathematical model of an aircraft performance parameters calculator

Essentially the mathematical model of an aircraft performance can be defined as list of mathematical expressions which can transform various numbers

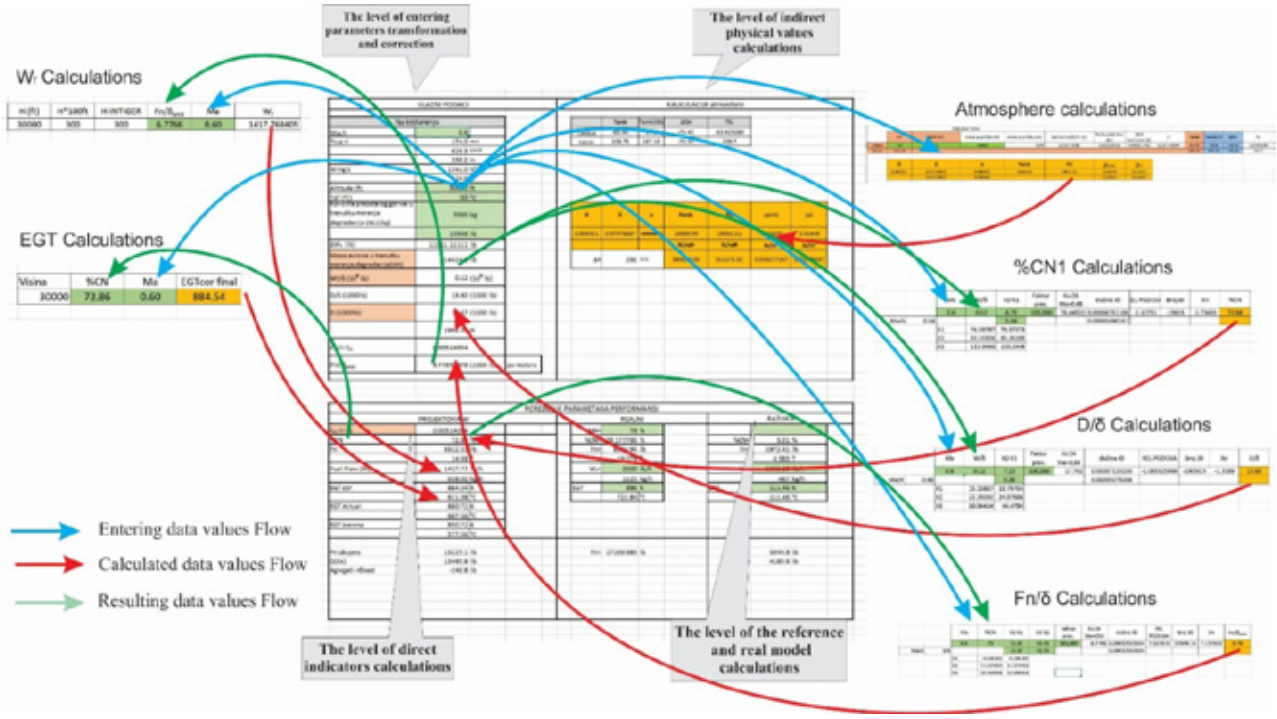

Figure 7. *Mathematical model of aircraft performance data calculated in Microsoft Excel.*

of entering data in required number of results and physical values which are necessary for the analysis.

Mathematical model of an aircraft performance parameters calculator has multilayer nature and cover different levels of calculations:

1. **The level of entering parameters transformation and correction –** which covers transformation of entering data (Ma, H, W, TAT) and its physical values in required values which will be used in further calculations.
2. **The level of indirect physical values calculations –** which implies entering data required by mid-calculations basically referred to defined real atmosphere model.
3. **The level of direct indicators calculations** – which covers the relations between the system of mathematical formulas and the entering and mid-calculated data which gives the set of resulting parameters required for the performance analysis.
4. **The level of the reference and real model calculations –** which will provide set of information data for the precise and complete performance indicators analysis.

In practical application of this technique an algorithm of required aircraft performance data will be defined (Figure 6.), and transformed in actual system of connected calculations (Figure 7.).

Conclusion

Energy efficient aircraft performance is corner stone of U-Space ATM development. Digitalization of aircraft nomograms and other experimental and testing data will provide further improvement of concepts and solutions in aircraft aerodynamical design.

In combination with time based navigation, digitalization of aerodynamical characteristic will be a step forward to introduction of Continuous Climb and Descent Trajectories, as the most optimal one.

As U-Space will be relatively smaller air-space volume, compared to the actual ones used for general traffic, with relatively short en-route phase, digitalized aircraft performance with time based navigation could provide environment for spherical or parabolic UTM system development, as a solution for requirements of dynamic U-Space Air-Traffic Management.

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