FUNCTIONAL DEPENDENCIES ANALYSE IN FUZZY RELATIONAL DATABASE MODELS

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Contribution to the State of the Art

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Abstract: This paper presents a literature overview of Fuzzy Relational Database Models with emphasis on the role of functional dependencies in logical designing and modeling. The aim is the analysis of recent results in this field. Fuzzy set theory is widely applied for the classical relational database extensions resulting in numerous contributions. This is because fuzzy sets and fuzzy logic are powerful tool for manilupating imprecise and uncertain information. A significant body of research in efficient designing FRDM has been developed over the last decades. Knowing the set of functional dependencies, database managers have a chance to normalize the same eliminating redundancy and data anomalies. In this paper we have considered the most important results in this field.

Key words: fuzzy relational database model, functional dependencies, fuzzy functional dependencies, fuzzy set.

INTRODUCTION

Classical database models often suffer from their incapability of representing and manipulating with imprecise and uncertain information that appear in many real world applications. Since the early ‘80s, Zadeh’s fuzzy logic has been used to extend different data models. The purpose of introducing fuzzy logic in the database is the possibility of representing and monitoring a vague and imprecise information. This resulted in numerous contributions, mainly in computer applications. Naturally, fuzzy relational database extends a function of classical data models, which provides a higher level of fuzzy system adaptation as one of the basic features of intelligent systems (in addition to system of planning, learning, prediction, system for knowledge search, robots). A very important thing of this data model is the fact that there are many active research areas that directly involve or use these knowledge base. The issue about vague and imprecise data and their representations is represented and important in various fields. We’ll list just a few of them: geographic information systems (GIS) and representation of spatial data systems, data mining, statistical database models, information retrieval.

In a relational database models real interest is the identification of dependencies between data, i.e. functional and fuzzy functional dependencies, so that these models could be normalized. In this way, the database design is based on the assumption that there is a set of dependencies which is the input for database normalization. There have been a lot of papers about data dependency analyse, but there isn’t comprehensive review in this area. In this paper, we have considered and systematically elaborated the concept of functional dependency that is extended to Fuzzy Relational Database Models (FRDM).

The remainder of this paper is organized as follows. Section 2. gives basic knowledge about fuzzy set theory and uncertain information. Fuzzy relational database models are described in section 3. Section 4. explores issues and papers in the field of functional dependencies analyze. The fifth section is scheduled for conclusion.

IMPERFECT INFORMATION AND FUZZY SET THEORY

Imprecise and unceratin information

Inconsistency, *imprecision*, *vagueness*, *uncertainty*, and *ambiguity* are five basic kinds of imperfect information in database systems [25].

a. Inconsistency is a kind of semantic conflict, meaning the same aspect of the real world is represented differently in one or in several different databases. For example, the *age* of one person is stored as 34 and 37 simultaneously.

b. Intuitively, the imprecision and vagueness are relevant to the content of an attribute value, which means that a atribute value must be made from a given range (interval or set) of values but we do not know exactly which value will be selected at present. In general, vague information is represented by linguistic variables. For example, the *young man* is a set {20,21,22,23} which means that a young man can be 20 or 23 years old.

c. The uncertainty is related to the degree of truth of its attribute value, and it means that we can apportion some, but not all, of our belief to a given value or a group of values. For example, the possibility that the *age* of *Marko* is 35 right now may be 97%. The random uncertainty described with probability theory is not considered here.

d. The ambiguity means that some elements of the data model lack complete semantics leading to several possible interpretations.

Generally, several different kinds of imperfection can co-exist regarding to the same data in database. For example, person's age is data from a set of values and their membership degrees are 0.85, 0.90, 0.96 and 0.80 respectively. Imprecision, uncertainty and vagueness are the most often types of imperfect information in classical relational database.

Fuzzy set theory and possibility distributions

Many of the existing approaches related to imprecision and uncertainty information are based on the theory of fuzzy sets and possibility distribution theory. A fuzzy set (0.85/20, 0.90/21, 0.96/22, 0.80/23) for the person's age contains uncertainty information (a person's age may be 20, 21, 22 or 23 years) and the degree of membership (0.85, 0.90, 0.96 and 0.80) simultaneously. One of the most important characteristics of fuzzy sets is their ability to express the degree of uncertainty in human thinking and his subjectivity. Such a basic idea with membership grade or weighted elements is proved as very useful in the knowledge analysis and information representation.

Let X be a domain. A fuzzy set A defined on X is usually displayed in the form:

$$\mu_A : A \rightarrow [0,1]$$

In this way, each element x in fuzzy set A has a degree of membership $\mu_A(x) \in [0,1]$. Thus the fuzzy set A is described as a set of n-tuples:

$$A = \{x, \mu_A(x) : x \in A\}$$

where $\mu_A(x)$ denotes the degree of membership of x in the fuzzy set A. When $\mu_A(x)$ is greater, there is more thruth in the claim that the element x belongs to A.

When X is an infinite set, fuzzy set A defined on X is represented as:

$$A = \int_x \frac{\mu_A(x)}{x}$$

Three major meanings for membership function which exist in the litareature are: similarity, preference and uncertainty. Each of these semantics can be used in real class of applications. Membership function of a fuzzy set is sometimes a kind of utility function that represents flexibile contstraints in the decision-making problems. In following paragraphs are defined interpretations of membership function using in applicatons.

Degree of similarity: membership function can be used for defining the degree of closeness and similarity between respective elements. This is also the oldest semantics introduced by Bellman et al. and this view is particularly significant in clustering and regression analysis where we have considered the problem of data representing and determining closeness between them.

Degree of preference: a fuzzy set A represents a set of more or less preferred objects or values of the variables using for decision making. In this case, $\mu_A(x)$ represents an intensity of preference in favour of object x as a value of y. Fuzzy sets then represent criteria or flexible constraints. This approach forwarded by Bellman and Zadeh is now fundamental for optimization problems, fuzzy linear programming and decision analysis. Approximate reasoning based on the variables and constraints that can be fuzzy is particularly suitable for using of this concept.

Degree of uncertainty: This interpretation is proposed by Zadeh when he introduced possibility distributions theory. $\mu_A(u)$ is the degree of possibility that a parameter X takes value u. Membership function ranks values in terms of their plausibility. This approach is used in expert systems. When the membership function is defined on this way then the probability that the parameter X takes value u describes as a possibility distribution p_X.

$$p_X = (p_X(u_1)/u_{1,}\, p_X(u_2)/u_{2,\dots,}\, p_X(u_n)/u_n)$$

Extension of classical relational database model introduced by Codd can be done by including fuzzy values on the attribute domain. These uncertainty information are defined by Zadeh's fuzzy sets and fuzzy logic theory and they allow mathematical framework for representation and handling of imprecise information in fuzzy relational database models.

Fuzzy Relational Database Models

Numerous studies in the field of fuzzy relational database models were introduced in recent years. The literature has reviewed and discussed various issues, such as data representation, different models of the fuzzy relational databases (FRDBMS), the dependence between data, normalization and implementation of FRDBMS and fuzzy query generation. In this paper we present a comprehensive overview of functional dependency analysis which plays an important role in the logical design and database implementation.

Data representation in FRDBMS

Several approaches that include fuzzy information adding in the relational database model are shown in the literature. So, at the first level, fuzzy relational database model is based on the similarity relation. The second group is FRDBM based on fuzzy relation. The most important approach utilizes possibility distribution. The existing approach at this level can be grouped in two classes: attribute value associated with the possibility distribution in the first case, while in another one n-tuple belongs to relation with grade of membership m.

Therefore, we must define a framework for representing imprecise information. Several extensions have been brought to the relational database model to capture the uncertain parts of the real world. This chapter presents four frames for fuzzy representation of data in FRDBMS [9-12]:

- basic framework based on similarity relation,
- basic framework based on possibility distribution,
- basic framework based on fuzzy relation and
- basic framework with extended possibility distribution.

Let R be the relation scheme **R**(Name, Address, Age, Productivity, Salary) and T1, T2, T3 instances at some point of time:

T1: (Mark, Boulevard revolution Str., {21,22,23}, good, high or medium)

T2: (John, New Belgrade, {0.7/22, 1 / 25, 0.8/28}, excellent, {low, medium})

T3: (Peter, Knez Mihail page, 27, satisfactory, high): μ, where m $\in$ [0,1].

In presented model, domain of the attributes can be linguistic terms (low, medium, high, satisfactory...), fuzzy sets (0.7/23, 1/25, 0.8/28), subsets of the given domain (low, medium). Then, we notice that n-tuple may belong to a given relation with some degree of membership.

The basic framework based on similarity relation (Buckles and Petry) provides that each domain of set of attributes in fuzzy relational databases is associated with similarity relation, rather than identity relation and value domain is defined as a subset of the basic set instead a one element as we can see in classical relation databases. Thus, we have following definition:

Definition 1. A fuzzy relation R is a subset of Cartesian product $2^{D1} \times 2^{D2} \times ... \times 2^{Dn}$ where D is finite domain and 2^{Di} is power set of $D_{i.}$ Any member of the relation is simply called tuple.

A fuzzy relational database is defined as a set of relations where each relation is a set of tuples. According that, fuzzy tuple t_i has the form $t_i= (d_{i1,} d_{i2},..., d_{in})$ where $d_{in} \subset D_i$, $d_{in} \neq \emptyset$.

Definition 2. A similarity relation is a mapping s_j: $D_j \times D_j \rightarrow [0,1]$ such that for x,y,z ∈ D_j :

$s_j(x,x) = 1$ (reflexivity)

$s_j(x,y) = s_j(y,x)$ (simmetry)

$s_j(x,z) > \max(\min(s_j(x,y), s_j(y,z))$ ***(max-min transitivity).***

For a given domain D_j, the threshold of similarity is defined as:

$$\text{Threshold (Dj)} = \min_{\forall i}\left\{ \min_{x,y \in d_{ij}}[s(x,y)] \right\}$$

We can present an example of this model with information:

T1:(Mark, Boulevard revolution Str., {21,22,23}, good, high or medium).

The basic framework based on possibility distribution extends the classical theory of relational databases allowing the use of fuzzy values for attributes. The fundamental concept of the fuzzy information is that a variable (attribute) is not definied as a specific value. In this context, we use the term possibility distribution where each attribute value is associated with values from the interval [0,1]. Generally, the possibility distribution is identified with membership function. And we say that element d belongs to a fuzzy set F ("Height of people") with degree of membership 0.9, then the possibility that variable X, defined on the domain F, takes the value d, π_x (d)=0.9.

Definition 3. A fuzzy relation R is a subset of the domain $\Pi(D_1) \times ... \times \Pi(D_n)$, where:

$\Pi(D_i) = \{\pi_{Ai} \mid \pi_{Ai}$ is possibility distribution of A_i on Di$\}$.

Corresponding tuple is given in the form $t_i= (\pi_{A1,} \pi_{A2,...,} \pi_{An})$.

Further, an extra-element e is introduced in this model which stands for the case when the attribute does not not apply to t_i. The possibility distribution can be viewed as a fuzzy restriction:

$\pi_{Ai:} D \cup e \rightarrow [0,1]$.

An example of this model is presented with information:

T2: (John, New Belgrade, {0.7/22, 1 / 25, 0.8/28}, excellent, {low, medium}).

The basic framework based on fuzzy relation is concept introduced by Baldwin. Fuzzy relation is defined as follows:

Definition 4. Fuzzy relation R on $D_1 \times ... \times D_n$ is determined by the membership function:

$\mu_R : D_1 \times ... \times D_n \rightarrow [0,1]$, where D_i is domain of attribute $A_{i.}$

General form of the binary relation R on $D_1 \times D_2$ is represented as:

$R = \{\mu_R(u_1,v_1)/(u_1,v_1),..., \mu_R(u_m,v_m)/(u_m,v_m)\}$ in tuple given as:

$R = \{u_1,v_{1,} \mu_R(u_1,v_1),...,u_m,v_{m,} \mu_R(u_m,v_m)\}$

where $u_j \in D_1$, j = 1,2,...,m and $v_k \in D_2$, k = 1,2,...,n.

This model specifies that a tuple belongs to a given relation with appropriate grade of membership μ, while the individual attribute values needn't be fuzzy or may be a linguistic variable, but they are treated as atomic or one-variable value.

An example of this model is presented with information:

T3: (Peter, Knez Mihail page, 27, satisfactory, high): μ, where $\mu \in [0,1]$.

The basic framework with extended possibility distribution extends the basic framework based on possibility distribution allowing not only distribution of attribute values, but also and proximity relation associated with a given domain. This extension generalizes the classical relational database model. Note that the similarity relations are only special proximity relations in which closeness relationships are reflexive and symmetric. The properties of reflexivity and symmetry are very appropriate for expressing the degree of closeness or proximity between elements of a scalar domain.

Definition 5. A proximity relation is a mapping $s_j: D_j \times D_j \rightarrow [0,1]$ such that for $x,y \in D_j$:

$s_j(x,x) = 1$ (reflexivity)

$s_j(x,y) = s_j(y,x)$ (simmetry)

In this way, the above-mentioned frameworks become special cases of the basic framework with extended possibility distribution.

An example of this model is presented with information:

T2: (John, New Belgrade, {0.7/22, 1 / 25, 0.8/28}, excellent, {low, medium}).

Gefred

In the previous years, some authors [1,2,13-16] have dealt with the issue of introducing imprecision and uncertainty information in relational databases. This leads us to the database systems which lie within the scope of artificial intelligence, because they enable to manage information which are very similar to natural language. Codd introduced the relational database organization that is based on relational theory. Zadeh's fuzzy set theory is a generalization of the general theory, while the fuzzy relation concept is generalization of the relational theory.

In this paper we review a general extension of the relational database model called GEFRED. Other models are considered as particular cases of this model. Group of authors [*] introduced General Fuzzy Relational Database Model (GEFRED) that incorporates elements of previous studies into a single model. In this section we introduce the basic elements of a fuzzy extension of relational model.

GEFRED structure model may be shown as follows:

$$R_{FG} \in (D_{G1,} C_1) \text{ x } \ldots \text{ x } (D_{GN,} C_n),$$

where D_{Gi} is a domain of attributes and C_i "attribute compatibility" which takes a value from the interval [0,1]. In this fuzzy relational model attribute compatibility values are not shown, but in each tuple, attribute value is associated with the appropriate value $C_{i.}$

Let us consider the following example that describes the extension of classical relational database model to GEFRED.

Table 1. Generalized Fuzzy Relational Database Model

Name	Address	Age	Productivity	Salary
Mark	Boulevard Revolution	31	Good	High
Alex	Medakovic	Middle	Satisfactory	10.000
Nes	Karaburma	Young	Bad	9.000
Smith	New Belgrade	Old	Excellent	Low
Volter	Cerak	Young	Good	Medium
Greg	Rakovica	About 28	Excellent	13.600
Mathew	Zarkovo	Between 30 i 35	Satisfactory	10.900

FIG. 1. MEMBERSHIP FUNCTION FOR THE ATTRIBUTE AGE [6]

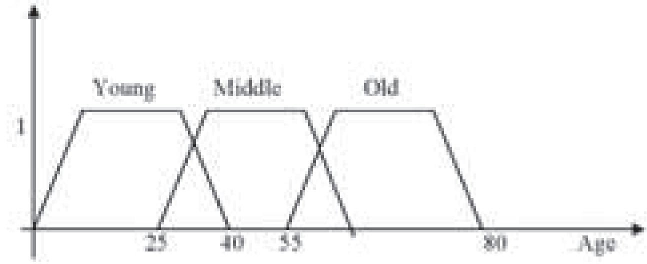
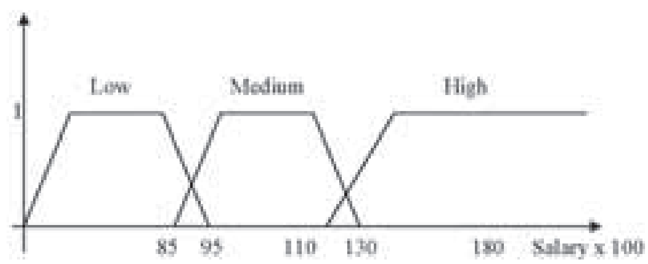


FIG. 2. MEMBERSHIP FUNCTION FOR THE ATTRIBUTE SALARY [6]



The attributes NAME and ADDRESS contain crisp information with the primary key attribute NAME. On the other hand, AGE and SALARY permit fuzzy information and corresponding membership functions for linguistic variables in the relation are shown in the Figs. 1 and 2. The attribute PRODUCTIVITY admists fuzzy information from a discrete domain and we need to define proximity relation over the elements of its domain. This is shown in Table 2.

TABLE 2. PROXIMITY RELATION OVER PRODUCTIVITY

$s(d_i,d_j)$	Bad	Satisfactory	Goog	Excellent
Bad	1	0.85	0.60	0.20
Satisfactory	0.85	1	0.70	0.55
Good	0.60	0.70	1	0.8
Excellent	0.20	0.55	0.80	1

This theoretical model includes all necessary elements for the definition of fuzzy relational database model (FRDBM). Within this database, we can examine the relationship between individual attributes (e.g. salary and productivity). We are able to implement a logical database model knowing functional dependencies. For this reason, in the next section we give a comprehensive overview of functional dependencies analyse between data and attributes in relations.

DATA DEPENDENCY

Integrity constraints play a key role in the logical database design. Among these limitations, data dependency is the most interesting one because it offers a direct possibility of normalization of relational database model. Therefore, special attention is dedicated to the study of functional dependencies. They bring into relation one set of attribute values with the values of another set of attributes. Based on different models of fuzzy relational databases, different approaches have been proposed for the expression of functional and fuzzy functional dependencies . We differentiate two types of papers in which these topics are studied: the first group includes papers in which the concept of fuzzy functional dependencies is defined, and the other group is consisted of papers in which the concept of functional dependencies for data decomposition and reduction of redundancy and approximation queries in the database is applied.

Functional dependencies

Definition 6. Let R be (A1, A2,..., An) the relational schema to the domains D1, D2,..., Dn with Dom (Ai) = Di and let X and Y be subsets of a set of attributes U = {A1, A2, ..., An} i.e. X, Y $\subseteq$ U and let r be the relation of R, r $\subseteq$ × ... × D1 Dn. We state that the relation r satisfies the functional dependency X → Y if for every two n – tuples t and t'$\subseteq$ r, for which t(X) = t' (X) applies, it is implied that it also stands for the t(Y) = t'(Y).

The above mentioned definition indicates that whenever the pairs (x, y) and (x, y ') are elements of R relation [XY] then y = y'. This is precisely the condition that distinguishes the function from the relation. If functional dependency X → Y does not exist, then the relation R [X, Y] can contain multiple elements that have the same attribute value of X, and different attribute value of Y. Secondly, X → Y is a time-invariably ability. A set of n - tuples that describe R (A1,..., An) changes in time, and it is valid for R [X, Y] as well. The definition of functional dependency requires that these changes are such that at any point of time R [X, Y] is not only a function but a relation R [X] → R [Y] as well. The importance of functional dependencies is reflected in the fact that through them we can determine the primary relation key and what is more important we can define a logical database model.

Fuzzy functional dependencies

If we include the functional dependencies in the fuzzy database model [1.26], the previous definition can not be directly applied to this model because it is based on the concept of equality. Since there is no clear way to verify when the two imprecise values are equal, then the definition of functional dependencies must be extended and generalized respectively. This extended / generalized version of the functional dependency is called the fuzzy functional dependency [1,3,5,15,17,20-22,36]. There are several different definitions of the fuzzy functional dependency, which are obtained as a result of the use of fuzzy logic in the classical functional dependencies and all such definitions of functional dependencies are associated with a given frame in the fuzzy database. Therefore, they are only applicable within a given framework, although there are basic and general features and characteristics that are required to have the fuzzy functional dependency.

In classical relational databases, functional dependencies determine when the value of the n-tuples of one set of X attribute uniquely determines its values in another set of Y attribute or strictly speaking: Generally, when the attribute values do not take on only the domain atomic elements, but also possibility distribution, then the X → Y degree is not necessarily required to be 1, but may be in the unit interval [0,1]. Therefore, the following questions are naturally imposed. How to determine the t (X) = t '(X) and t (Y) =t' (Y) when t (X), t '(X), t (Y) and t' (Y) are all imprecise values of possibility distribution. Secondly, how to determine the level of propositions; if t (X) = t '(X) then t (y) = t' (Y) where t (X) = t '(X) and t (Y) = t' (Y) are partially true with the degree of membership from the interval [0,1]. Finally, how to assess the degree of X → Y if different pairs of n - tuples give different true values for the proposition if ... then. Hence what arises is that these issues are associated with problems of fuzzy proximity data, fuzzy logic implications and fuzzy (and) operator. Now, let us define the fuzzy functional dependencies in the following way.

Definition7. Let R be (A1, A2,..., An) the relational schema to the domains D1, D2,..., Dn with Dom (Ai) = Di and let X and Y be the subsets of a set of attributes U = {A1, A2, ..., An} i.e. X, Y ⊆ U and let r be the relation of R, r ⊆ Π (D1) × ... × Π (Dn), where Π (Di) = {π | π is a possible distribution of Ai at Di, i = 1,2,..., n.

We say that X fuzzy functionally determines Y with the degree of θ designated as $X \overset{\theta}{\rightarrow} Y$ if and only if for ∀ r ∈ R:

min I (t (X) = c t '(X), t (Y) = c t' (Y)) ≥ θ, t, t '∈R.

where ∈θ [0,1] = C [0,1] × [0,1] → [0,1] is a measure of proximity (closeness)

and I: [0,1] × [0,1] → [0,1] is a fuzzy implication operator.

The rules of executing functional dependencies

In classical relational model, it is often necessary, based on a given set of dependent data, to determine some other dependencies on the same database which are the result of already given set of dependencies. Therefore, in the classical relational databases, there are three rules of executing known as Armstrong's axioms, and which are used to derive new functional dependencies from the given functional dependencies. Now, we provide these rules of executing [9]. Let X, Y, Z and W be an arbitrary set of attributes.

P_1: If $Y \subseteq X$ then the functinal dependency exist X → Y.

P_2: If functional dependency X → Y applies then the functional dependency XZ → YZ also applies.

P_3: If functional dependencies X → Y and Y → Z apply then the functional dependency X → *Z also applies.*

Fuzzy functional dependencies express the relation between the set of attributes. If the relational schema is given to a set of attributes U and the fuzzy functional dependency $X \xrightarrow{\theta} Y$, which is satisfied in R, then $X \xrightarrow{\theta} Y$ signifies that it is satisfied in all the relations of R. Moreover, for a given set of fuzzy functional dependencies F, which is satisfied in F, does not only guarantee that these fuzzy functional dependencies of F are met in all relations of R, but it also guarantees that each fuzzy functional dependency, logically implied with F, is satisfied in all relations of R. For example, if we know that $X \xrightarrow{\theta} Y$ in R applies, then we can expect that in R also applies $X \xrightarrow{\alpha} Y$ for a ≤ θ from [0,1]. Also, if $X \xrightarrow{\theta} Y$ applies then we can expect that XZ functionally determines YZ to a degree that is at least equal to θ, for all relations of R. Furthermore, if $X \xrightarrow{\alpha} Y$ and $Y \xrightarrow{\beta} Z$ apply in R then we can expect that X functionally determines Z with some degree of λ from [0.1]. Intuitively, we can assume that λ= min (a, β) for the following reasons:

a. If X functionally determines Y with the degree of θ and Y functionally determines Z with the degree of θ, then we can expect that X functionally determines Z with the same degree.

b. If X functionally determines Y with the degree of $\alpha \geq \theta$ and Y functionally determines Z with the degree of $\beta \geq \theta$, then we expect that X functionally determines Z with the degree of θ. Thus, the above given three Armstrong's axioms in the classical relational theory are expanded with three rules of executing fuzzy functional dependencies:

   P'1: If $Y \subseteq X$ then there is fuzzy functional dependency $X \xrightarrow{\theta} Y$ for ∀θ.

   P'2: If fuzzy functional dependency $X \xrightarrow{\theta} Y$ applies, then the fuzzy functional dependency $XZ \xrightarrow{\theta} YZ$ also applies.

   P'3: If fuzzy functional dependencies $X \xrightarrow{\alpha} Y$ and $Y \xrightarrow{\beta} Z$ apply, then the fuzzy functional dependency $X \xrightarrow{\lambda} Z$ *with* λ = *min* (α,β), where λ, α, β, ∈θ [0,1].

Analysis models of functional dependencies

In the field of functional and fuzzy functional dependencies we point out the following papers [2,11,13,14,21, 23,24,30,32-35].

Fuzzy functional dependencies in Raju and Majumdar's model

Raju and Majumdar's [30] fuzzy relational model allows components of n- tuples to take both atomic and non-atomic values. Depending on the complexity of the domain, they divide fuzzy relation into two categories. In the first type of the fuzzy relation, domain can only be a fuzzy set or classical set. The second type of the fuzzy relation ensures that each attribute domain can be a classical set, fuzzy set or a set of fuzzy subsets (or possibility distribution). Each fuzzy relation in this model is represented by a table that has an additional column which determines the membership value of a given n – tuples to an appropriate relation.

Raju and Majumdar define the fuzzy functional dependencies in the following way:

Definition 8. Fuzzy functional dependency X → Y, X, Y ⊆ R, applies in fuzzy relation r on R, if for all n-tuples t_1 and t_2 from r ($m_r(t_i) > 0$, i = 1,2), applies:

$$\mu_{EQ}(t_1[X], t_2[X]) \leq \mu_{EQ}(t_1[Y], t_2[Y])$$

where Equal (EQ) is a fuzzy relation of proximity (closenesss) on a universal set U and is defined as a fuzzy subset on U × U and where μ_{EQ} is a membership function that satisfies the following conditions: For every a,b ∈ U, $\mu_{EQ}(a,a) = 1$ (reflexive) and $\mu_{EQ}(a,b) = \mu_{EQ}(b,a)$ (symmetric).

Fuzzy functional dependencies in Saxena and Tyagi's model

In the fuzzy relational model of Saxena and Tyagi [32] fuzzy attribute values are allowed the possibility where the attribute is not applicable to a given object. To be able to work with vague data values, they define the fuzzy relation as follows. Let 2^{Ai}, i =

1,2,...,n be a set of fuzzy subsets on the domain dom $A_i \cup \{e\}$, where e is the extra element which allows the opportunity for a fuzzy attribute value not to be applied to a given object, such that for each set of $a \in 2^{Ai}$, a membership function from a satisfies the condition $\mu_a(e) = 0$ ili $\mu_a(e) = 1$ of the set with $\mu_a(u) = 0$ for each u. At that point, the fuzzy relation r on $R = (A_1 ... ,A_n)$ is defined as a fuzzy subset of the Descartes' product $2^{A1} \times 2^{A2} \times ... \times 2^{An} = 2^R$, characterized by a membership function:

$$\mu_r: 2^R \rightarrow [0,1].$$

Every n - t = tuple $t = (a_1,...,a_n)$ where $a \in 2^{Ai}$, i = 1,2,...,n in r can be seen as possibility distribution for $D = (\text{dom } A_1 \cup \{e\}) \times \ldots \times (\text{dom } A_n \cup \{e\})$ as:

$$Poss(t[A_1] = u_1, ..., t[A_n] = u_n) = \min \{\mu_r(t), \mu_{a1}(u_1),..., \mu_{an}(u_n)\},$$

where $u_i \in A_i, \cup \{e\}$, i = 1...n.

In this fuzzy relational model, the similarity between the elements of a given domain is defined as follows.

$$\mu_{EQ}(a_1,a_2) = \min_{u \in domA_i} \psi(\mu_{a1}(u), \mu_{a2}(u))$$

Here the fuzzy relation is EQ na 2^A-{Ø}, where, with Ø it is marked that for the attribute A, a fuzzy set $a \in 2^A$ is such that $\mu_a(e) = 1$ *and* $\mu_a(u) = 0$ is *for* $u \neq e$, and is defined as a fuzzy subset $(2^A\text{-}\{Ø\}) \times (2^A\text{-}\{Ø\})$ so that its membership function is defined as $\mu_{EQ}(2^A\text{-}\{Ø\}) \in (2^A\text{-}\{Ø\}) \rightarrow [0,1]$ and which meets the $\mu_{EQ}(a,a) = 1$ (reflexive), $a \in 2^A\text{-}\{Ø\}$ and $\mu_{EQ}(a_1,a_2) = \mu_{EQ}(a_2,a_1)$ (symmetric), $a_1, a_2 \in 2^A\text{-}\{Ø\}$. ψ is the fuzzy relation of similarity. Based on the above considerations, in this model, they introduce fuzzy functional dependencies.

Definition 9. Fuzzy functional dependency X → Y, X, Y ∈ R, applies in fuzzy relation r on R, if for any n-tuples t_1 and t_2 from r $\mu_r(t_i) > 0$, i = *1,2* applies $\mu_{EQ}(t_1[X],t_2[X]) > 0$, $t_1[Y] = t_2[Y] = Ø$ or a non-empty set exists Y '⊆ Y such that $t_1[A] \neq Ø \neq t_2[A]$ for every A∈Y', $t_1[Y\text{-} Y'] = t_2[Y\text{-}Y'] = Ø$ *and* $\mu_{EQ}(t_1[X], t_2[X]) \leq \mu_{EQ}(t_1[Y'], t_2[Y'])$.

Fuzzy functional dependencies in Wei-Yi Liu model

This paper first defines the concept of semantic distance between two fuzzy values of attributes, while the fuzzy functional dependencies are represented by the fuzzy semantic distance. Based on the semantic proximity, definition of fuzzy functional dependencies is given [21,23,24]. The degree of closeness between the two fuzzy values is described by means of semantic proximity. Semantic proximity is based on the concept of the interval and we mark it with the $SD(f_1,f_2)$, where $0 \leq SD(f1, f2) \leq 1$. The following characteristics should be met:

If $f_1=[a_1,b_1]$, $f_2=[a_2,b_2]$, $g_1=[c_1,d_1]$, $g_2=[c_2,d_2]$. Then applies:

1. $SD(f_1,f_2)=1$ if and only if $a_1 = a_2 = b_1 = b_2$,

2. $SD(f_1,f_2)=0$ if and only if $f_1 \cap f_2 = Ø$,

3. If $a_1 = a_2$, $b_1 = b_2$, $c_1 = c_2$, $d_1 = d_2$ and $|d_1\text{-} c_1| > |b_1\text{-} a_1|$ then $SD(f_1,f_2) \geq SD(g_1,g_2)$,

4. If $|a_2\text{-} b_2| = |a_1\text{-} b_1|$ and $f_1 \cap g_1 \geq f_2 \cap g_1$ then $SD(f_1,g_1) \geq SD(f_2,g_1)$.

Semantic distance (proximity) is calculated by the following formula:

$$SD(f_1,f_2)=||f_1 \cap f_2|| / ||f_1 \cup f_2|| - ||f_1 \cap f_2|| / \alpha$$

where ||h|| is modular with an appropriate interval:

$$\|h\| = \begin{cases} 0 & h \neq 0 \\ \delta & h = [a,a] \\ |b - a| & h = [a,b] \\ \alpha & h = \infty \end{cases}$$

α is such a coefficient that $\alpha \geq ||f_1 \cup f_2||$, and δ is a small number. E.g. δ=α/10.000 is usually taken in concrete examples.

If $t_1= (x_{11}, x_{12}, ..., x_{1n})$ and $t_2= (x_{21}, x_{22}, ..., x_{2n})$ are two n-tuples in relation, then the semantic proximity between them is marked as $SD(t_1,t_2)$ and calculated as $SD(t_1,t_2)=\min\{SD(x_{1i},x_{2i})\}$.

The semantic distance $SD(f_1,f_2)$ of the two fuzzy values f1 (X) and f2 (X) is defined by using a certain standard $|f_1(X) - f_2(X)|$. For example, $SD(f_1,f_2) = \max |f_1(X) - f_2(X)|, x \in dom(A_i)$. The complement of $SD(f_1,f_2)$ in the $SS(f_1, f_2)$ mark is defined as $SS(f_1, f_2) = 1 - SD(f_1, f_2)$. The semantic distance of the fuzzy values described in another way can be defined similarly.

Definition 10. Let r be the fuzzy relation on the relational schema $R(A_1,...,A_n)$ and let U be a universal set of attributes $A_1,...,A_n$ and let X and Y be subsets of U. We say that fuzzy relation r meets fuzzy functional dependency $X \rightarrow Y$, $X,Y \subseteq R$ if for each pair of n-tuples t_1 and t_2 in r applies:

$$SS(t_1[X], t_2[X]) \leq SS(t_1[Y], t_2[Y])$$

Fuzzy functional dependencies in Dubois-Prade model

Dubois and Prade [13,14] introduce fuzzy functional dependencies as follows: if the attribute A values for the n-tuples t and t' are equal then the attribute B values for t and t' should not be far away from each other. They model this idea by expressing the fuzzy relations of closeness P (which is reflexive $\forall d, \mu_p(d,d) = 1$ and symmetric $\forall d,d' \; \mu_p(d,d') = \mu_p(d',d)$), and which is defined in the field of attribute B meaning.

If $t(A) = t'(A)$ then $\mu_B (t(B),t'(B)) > \theta$

where θ represents inflicted threshold.

Considering that t(B) i t'(B) is determined in the term of possibility distribution fuction, we have:

If $t(A) = t'(A)$ then $\Pi (t(B) \approx_p t'(B)) > \theta$

This possibility is given with:

$$\Pi (t(B) \approx_p t'(B)) = \sup_{v,w \in D_B \times D_B} \min (\mu_p(v,w), \pi_{t(B)}(v), \pi_{t'(B)}(v))$$

where π_B is a function of possibility distribution which limits possible meaning of the attribute B for the n-tuple t, and μ_p is a membership function of the closeness P fuzzy relation.

Fuzzy functional dependencies in Shenoi-Melton model

Their strategy for the expression of imprecise information is based on the idea of collection of congenial elements of the final domain into blocks of elements that do not differ from a certain level of accuracy in that domain. This idea is expressed in relation to the classes of equivalence at the domain of partition. The partition of the domain Dk is the set of non-empty disjoint subsets or equivalence classes of Dk with the property that each element from Dk is exactly in one equivalence class. [33.34] Partition of the domain into classes of equivalence is the key to the preservation of some important features in the classical model. On this basis they define redundancy as follows:

Definition 11. Let t and t' be two fuzzy n-tuples. Components t_k and t'_k are α_k – redundantly marked as $t_k \approx_{\alpha k} t'_k$, when t_k and t'_k are subsets of the same equivalence class for α_k – partititions of the temporal domain D_k.

They define fuzzy relation as follows:

Definition 12. Let R be the relational schema with attributes $(A_1...,A_n)$ and adjoint partitions with levels $(\alpha_1,..., \alpha_n)$. Let r be $(R) \subseteq 2^{D1} \times 2^{D2} \times ... \times 2^{Dn}$. Then r is the fuzzy relation in relation to the fuzzy relational schema R and temporal domain $D'_1,...,D'_m$, if r is a set of non-redundant fuzzy n-tuples with respect $\alpha_1,..., \alpha_n$ – partition level on $D'_1,...,D'_m$, respectively.

Definition 13. Let R be the fuzzy relational schema with attributes U and level of partition of α_U. Let X and Y be subsets of attributes in U with associated levels $\alpha_X = (\alpha_p,..., \alpha_q)$ and $\alpha_Y =(\alpha_r,..., \alpha_s)$ in α_U. Let r be the fuzzy relation with temporal domains $D'_p,...,D'_q$, and $D'_r...,D'_s$, for subsets of attributes X and Y respectively. Relation r meets the fuzzy functional dependency from X in Y with levels (α_X,α_Y) on given partitions, when for any n-tuples $t, t' \in r(R)$ applies:

t $\approx_{\alpha x}$ t' implies t $\approx_{\alpha y}$ t'

Finally, they argue that such defined fuzzy functional dependencies satisfy Armstrong's axioms.

Fuzzy functional dependencies in Sozati and Yazici model

One of the most fundamental definitions of the fuzzy functional dependencies is given in paper [35]. The analysis of the fuzzy functional dependencies is based on the following considerations. If t [X] is similar to t '[X], then t [Y] is also similar to the t' [Y]. In fact the similarity between the Y values is greater than or equal to the similarity between the X values. Such dependency is marked as $X \underset{F}{\rightarrow} Y$.

An example of such dependency is a dependency "officers with similar experiences should have similar salary".

In this case, attribute values of Experience and Salary can be imprecise, while the definition of dependency is strictly defined. However, this definition of functional dependency is not fully determined, in the sense that the dependency itself can be imprecise. An example of such functional dependency is a "level of person's intelligence *more or less* determines his/her success", where *more or less* in the sentence determines imprecise dependency. If we know that the person is intelligent, then we can conclude that he/she will be successful. However, this level of success is not clearly nor precisely determined by intelligence. A person can be very successful, less successful, and so on. Thus, this dependency does not determine the precise level of success, but at least it ensures a minimum level of success. Suppose there are two people with an identical intelligence and suppose that the first person is very successful. From this, one can not conclude that the other person is very successful, but we can say that the level of success of the other person will be more or less similar to the level of success of the first person.

One way to define this kind of dependency is to accept the linguistic intensity of dependency as a threshold. For example, dependency "officers with similar experiences should have similar salary" has a linguistic strength of 1, while dependency "The level of intelligence more or less determines the success" has a linguistic strength of 0.7. Thus, the threshold value determines the dependency intensity (strength) and it is written in the form of $X \overset{\theta}{\underset{F}{\rightarrow}} Y$, where θ is the dependency intensity (strength). The concept of similarity is very important in fuzzy relational databases because it allows us to extend the concept of identity into a clear model for handling imprecise and uncertain information. In "Crisp" models of databases, two n-tuples are identical on the observed attribute if and only if the values of that attribute are identical. In the fuzzy models of databases, the similarity of attribute values is observed in the sense to which extent these values are adjusted on the observed attribute. In this fuzzy model, the similarity between the attribute values is defined as the conformance of the two n-tuples on the attribute. This is very important aspect proposed by Bosc and others which is used for the comparison of the values of imprecise, fuzzy attributes by using the concept of conformance. The conformance relation is symmetric, reflexive and transitive.

Definition 14. The attribute conformance A_k is defined on the domain D_k for any n-tuples t_i and t_j, presented in relation r and marked as $C(A_k[t_i,t_j])$, is given as:

$$C(A_k[ti,tj]) = \min\left\{\min_{x\in d_i}\left\{\max_{y\in d_j}\{s(x,y)\}\right\},\min_{x\in d_j}\left\{\max_{y\in d_i}\{s(x,y)\}\right\}\right\},$$

where d_i is the attribute value of A_k for n-tuple t_i, d_j is the attribute value of A_k for n-tuple t_j, s(x,y) is the similarity relation for values x and y, and s is a mapping of each pair of elements from the domain D_k in the interval [0,1],

If $C(A_k[t_i,t_j]) > \theta$, for relation r, for n-tuples t_i and t_j we state that they are agreeable on the attribute A with the dependency intensity of θ. This definition is extended to the description of closeness for two n-tuples on the set of attributes.

Definition 15. The conformance on the set of the attribute X for any two n-tuples t_i and t_j, given in relation r, marked as $C(X[t_i,t_j])$, is given as:

$$C(X[t_i,t_j]) = \min_{A_k\in X}\{C(A_k[t_i,t_j])\}$$

Definition 16. Let r be the fuzzy relation on the relational schema $R(A_1,...,A_n)$ and let U be a universal set of attributes $A_1,...,A_n$ and let X and Y be the subsets of U. We state that the fuzzy relation r meets the fuzzy functional dependency $X \xrightarrow[F]{\theta} Y$, if for each pair of n-tuples t_i and t_j in r applies:

$$C(Y[t_i,t_j]) \geq \min(\theta, C(X[t_i,t_j]))$$

where θ is a realistic number from [0,1] and describes the linguistic strength of dependency.

Fuzzy functional dependencies in Cubero – Medina model

The theory of normalization which has been introduced by Codd is a systematic approach to a proper designing of databases. The main idea is that if we are faced with relations in databases which satisfy functional dependency (excluding the primary key), then there is a possibility of redundancy and updating of the existing base. In order to avoid redundancy, we can decompose the original relation, i.e. create decomposition, without having lost any information. Normally, in real databases, it is not usual that strict dependencies in relations are given. Nevertheless, we can find functional and fuzzy functional dependencies such as „The weight of a person more or less depends on his/her height and age". In these situations, the process of decomposition is proposed [11] and extraction of information respecting given dependency and compression of original data in relational database. The idea represents the use of fuzzy set theory and tolerance towards some uncertainties in the base, which allows us to include more n-tuples into one. Let us consider for example the relation which appears in the following relation. A special association (mapping) operator is used for the recovering of original data from the R relation.

Table 3. Original relation R [11]

X	Height	Weight
X1	180	86
X2	170	74
X3	170	73

Table 4. The introduction of fuzzy values in relational structure[11]

X	Height	Weight
X1	High	ca 85 kg
X2	170	74 kg
X3	170	ca 73 kg

Table 5. Relation r_1 [11]

X	Height
X1	180
X2	170
X3	170

Table 6. Relation r_2 [11]

Height	Weight
High	ca 85 kg
170	74 or ca73 kg

The decomposition of the relation R is given by relations (projections) r1 and r2, as shown in the previous tables. As can be seen in this example, we have reduced redundancy because the second and third tuple are merged into one in relation r2. In such situations, it is of primary importance to quantify how much of the imprecision we can tolerate, in order to guarantee that the fuzzy values such as "Ca 85 kg" are close enough to the original data of 86 kg. In order to do this, we must use the measure of similarity between data elements. For the new data in the base, we can test the fuzzy dependency by observing n-tuples in relation r2. In this way, the amount of data stored in databases r1 and r2 is smaller than in relation R. The original data that appears in R can be obtained by merging of relations r1 and r2. It should be noted that linguistic variables, such as "Ca 85 kg" are defined and given by the experts - experienced database managers. Therefore, we should be able to discover some knowledge in the form of the fuzzy rules that will allow us, that after decomposition of the original relation, we reduce and remove redundancy. This certainly allows us to get a better understanding of the real world, because fuzzy dependencies are isolated in a special relation.

The definition of fuzzy functional dependency in the Cubero – Medina model:

Definition 17. If $R_i(X_i(t_1),X_i(t_2)) \geq \alpha_i$ $\forall$i then $R_j(Y_j(t_1),Y_j(t_2)) \geq \beta_j$ $\forall$j must apply. As a special case we have $\forall$ $t_1, t_2 \in r$ where apllies that $X_i(t_1)=X_i(t_2)$ $\forall$i, then $R_j(Y_j(t_1),Y_j(t_2)) \geq \beta_j$ $\forall$j, for the case of the existence of fuzzy functional dependencies.

So, Cubero, Medina and others introduce other fuzzy functional dependencies in the relational database model. This approach allows us to discover connections between the attributes that are not detected by the classical approximation, and execute decomposition respecting established fuzzy dependencies. In this way, we reduce the redundancy in the databases, save computer resources, while at the same time there is no loss of information. The disadvantage of this approach is the fact that for efficient design and establishment of the fuzzy dependencies the help of experts and experienced database managers is necessary.The solution for this kind of problem could be data mining systems, i.e. the design of efficient algorithms for detecting the fuzzy functional dependencies without involving subjective human factor.

Fuzzy approximate dependencies

Within the analysis of data in relational databases, a very interesting question is detecting possible relations between attribute values, and at a higher level the relation between the attributes themselves, respectively, i.e. the analysis of functional and multivalued dependencies. In the case of the presence of uncertainty and vagueness of data, specific methods of data mining techniques are used in knowledge discovery. Berzal, Blanco and others [2] propose an algorithm for computing approximate fuzzy dependencies and different types of relations between attributes in the fuzzy relational database models.

In real databases, we are faced with two various types of relations. On the one hand, there are relations that are implicit, in which the relations between the attribute values are hidden and which are not clear enough at first. This type of dependency is obtained through the analysis of the database itself. On the other hand, we are often faced with the explicit relations between attributes that are easily detected (e.g. City and Zip Code). These two types of relations between attributes in the relational database structures represent integrity constraints that are imposed in the process of database design. In these cases we argue that there is a functional dependency or approximate dependency between attributes.

Search for functional dependencies in relational databases is a subject of interest in the field of data mining, as this form of business intelligence strictly deals with the structure of data. However, it is very difficult to perfectly detect functional dependencies in databases as a single exception in the rules affects the loss of dependency. If a number of these exceptions is not large, ''fuzzy functional dependencies with exceptions'' can indicate interesting regularities contained in the data. Moreover, the level of dependency which exists between the data is determined and presented. The idea is to measure not only the accuracy of dependency, but also support (the proportion of n-tuples in which the observed dependency occurs). Therefore, for the dependency assessment, Confidence is used - the conditional probability p (Y / X), written as Conf $(X \rightarrow Y)$ and support (Support) - The probability $p(X \cup Y)$, written as S $(X \rightarrow Y)$.

The problem with Confidence is the fact that it does not take into account the negative dependencies, therefore, high percentages of confidence can be obtained, which in these cases can be misleading. Therefore, in papers, the use of the safety factor CF is proposed:

$$CF(X \rightarrow Y) = \begin{cases} \frac{Conf(X \rightarrow Y) - S(Y)}{1 - S(Y)}, & Conf(X \rightarrow Y) > S(Y) \\ \frac{Conf(X \rightarrow Y) - S(Y)}{S(Y)}, & Conf(X \rightarrow Y) < S(Y) \\ 0 & Inace \end{cases}$$

Safety factor takes values from the interval [-1,1] and shows us to which extent is our conviction of the dependency existence true. CF= 1 in situations where X =True then Y = True, and CF =- 1 otherwise. Two extreme cases are when S (Y) = 0 and S (Y) = 1. In both cases the result is trivial, therefore it is logical that we then take the value of CF = 0.

Definition 18. If CF $(X \rightarrow Y) = 1$ (which implies that Conf $(X \rightarrow Y) = 1$) then $X \rightarrow Y$ is a functional dependency.

CONCLUSION

Inclusion of fuzzy information in different models of databases is an important research topic because fuzzy data is intensively present in a number of applications that we face and work with. The very essence of this kind of data models is the fact that there are many active research areas that directly include or use these knowledge bases, such as: geographic information systems (GIS) and spatial data representation, data Mininig systems, fuzzy information search, the statistical database models.

In this paper, a survey of different approaches in the analysis of functional dependencies which play a key role in designing and logical database designing has been conducted. Various fuzzy models based on the analysis of data dependency have been proposed in the last two decades and there is a significant number of papers and a large number of authors who deal with this issue. As we have seen, there are several frameworks for defining the functional and fuzzy functional dependencies, and which are more or less based on the similarity relation between the elements of a given domain. For all of them there are appropriate rules of executing, which demonstrate when from a given set of fuzzy dependency (functional or fuzzy functional) other dependencies are logically derived. In order for the new fuzzy dependencies to be derived from a given set of fuzzy dependency, there must be appropriate axioms made for them, which are based on Armstrong's axioms for the classical dependency. All these dependencies and rules of deduction must satisfy the adequacy requirement (sound) and completeness (complete).

However, it is noticed that the test dependency procedure and derivation of the logical consequences from a given set of attributes is very complex. Practically, there is no efficient algorithm that would enable us to easily identify the dependencies between the observed set of attributes and application of the normalization theory. Therefore, the subject of future studies is defining the framework and application of different mathematical tools that will enable simpler identification and discovery of knowledge necessary for the elimination of redundancy and different types of anomalies.

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